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Conjunctive item response models are introduced such that (a) sufficient statistics for latent traits are not necessarily additive in item scores; (b) items are not necessarily locally independent; and (c) existing compensatory (additive) item response models including the binomial, Rasch, logistic, and general locally independent model are special cases. Simple estimates and hypothesis tests for conjunctive models are introduced and evaluated as well. Conjunctive models are also identified with cognitive models that assume the existence of several individually necessary component processes for a global ability. It is concluded that conjunctive models and methods may show promise for constructing improved tests and uncovering conjunctive cognitive structure. It is also concluded that conjunctive item response theory may help to clarify the relationships between local dependence, multidimensionality, and item response function function form. (Author)

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CONJUNCTIVE ITEM RESPONSE THEORY KERNELS

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Conjunctive item response models are introduced such that (a) sufficient statistics for latent traits are not necessarily additive in item scores; (b) items are not necessarily locally independent; and (c) existing compensatory (additive) item response models including the binomial, Rasch, logistic, and general locally independent model are special cases. Simple estimates and hypothesis tests for conjunctive models are introduced and evaluated as well. Conjunctive models are also identified with cognitive models that assume the existence of several individually necessary component processes for a global ability. It is concluded that conjunctive models and methods may show promise for constructing improved tests and uncovering conjunctive cognitive structure. It is also concluded that conjunctive item response theory may help to clarify the relationships between local dependence, multidimensionality, and item response function form.

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Introduction

Research during the last decade has led to increased generality in item response theory. Important results include multidimensional item response models (Embretson, 1984; Fischer, 1973; McKinley & Reckase, 1983), locally dependent models (Kempf, 1977; Fischer & Formann, 1982), tests for unidimensional and locally independent models (Holland, 1981; Kelderman, 1984; Molenaar, 1983; Rosenbaum, 1984; Yen, 1981), and attempts to distinguish local dependence from multidimensionality (Goldstein, 1980; Hambleton, Swaminathan, Cook, Eignor, & Clifford 1978; Kelderman, 1984).

Applications of extended item response theory have also appeared, notably in componential reasoning models (Sternberg, 1977; Whitely, 1980). The componential approach involves breaking down each item reflecting a global ability or trait into subtask items. The subtask items presumably reflect component abilities/traits that are individually necessary for having the global attribute. Statistical procedures are then used to fit multidimensional item response models to subtask performance measures (Embretson, 1984; Fischer & Formann, 1982; Whitely, 1980).

Componential cognitive models as well as other serial process models (e.g., Anderson, 1981) imply a nonadditive combination rule for items that reflect distinct, individually necessary ability components. In particular, if binary items were used to reflect such components then the global trait would be reflected only if all component items were passed, as indicated by a product of item scores rather than a weighted sum. Parallel processes, on the other hand, would imply that items should be combined additively. Thus, serial process models are conjunctive in that all components are necessary for the process to occur, as potentially reflected by

multiplicative composites of item scores. In contrast, parallel processes are compensatory (Embretson, 1985) in that deficits in one component ability could be overcome by strengths in another, as reflected by additive composites of item scores. Moreover, additive item combination rules may be inappropriate for reflecting conjunctive/serial processes, whereas multiplicative item combination rules may be inadequate for reflecting compensatory/parallel processes.

Despite the above-cited recent advances in latent trait theory, no latent trait models have yet appeared with the potential for reflecting serial processing in standard paper and pencil tests. Standard test theory models including the Rasch, logistic, binomial, and classical models (Lord & Novick, 1968) lack this potential, because they imply sufficient statistics for latent traits that are additive in the item scores. Other special-purpose models have appeared for certain tests that are inherently conjunctive (Embretson, 1984; Stegelmann, 1983; Fischer & Formann, 1982). However, no general models currently exist for uncovering and reflecting conjunctive processes underlying standard tests. This is especially noteworthy given the prominence of serial process models in modern cognitive psychology.

Another existing gap in latent trait theory lies in the class of available locally dependent models. Although methods are available for detecting local dependence (Kelderman, 1984), weak variants of local independence have been formulated (Holland, 1981), and specialized locally dependent models have appeared (Embretson, 1984; Fischer & Formann, 1982: Kempf, 1977), no general family of locally dependent latent trait models has yet been introduced. (One general latent class model has been introduced, however--see Harper, 1972.) Furthermore, the lack of locally dependent models is directly connected with the lack of nonadditive models for conjunctive cognitive structure, as will be shown below.

The main purpose of this article is to provide a variety of kernels for item response models that reflect both conjunctive cognitive structure and local dependence. (The conditional distributions appearing in observable probability mixtures for these models are called kernels.) An auxiliary purpose is to demonstrate the utility of related procedures for estimating conjunctive item parameters and testing specific conjunctive/locally dependent structures.

Before describing conjunctive item response theory, the substance of conjunctivity and its connection to some related work will be illustrated.

Conjunctivity and configural scoring. For simplicity, suppose that a global trait (θ), say analogical reasoning ability, were tied to two individually necessary component abilities, say vocabulary (V) and inductive reasoning (I). Thus, items that were easy with respect to V yet moderately difficult with respect to I might reflect I alone, whereas items with the converse property might reflect V alone. If there were a few such pure items in an analogical reasoning test with the remaining items reflecting the global trait, then the test could be represented by Figure 1. As Figure 1 suggests, an optimal estimate of θ involving raw scores on x_3 through x_M might also involve x_1x_2, because passing both x_1 and x_2 is similar to passing each of the others, whereas passing either x_1 or x_2 alone is not.

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Insert Figure 1 about here

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More generally, a collection of M binary items may involve several conjunctive components. In this case an optimal estimate for θ might take the form,

$$\tilde{\theta} = t(\sum_{G_M} x_{m_1} \ldots x_{m_s} \alpha_{m_1 \ldots m_s}), \qquad (1)$$

where G_M is the power set ($G_M=\{(m_1,\ldots,m_s) \subseteq \{1,\ldots,M\};\ s=1,\ldots,M\}$--the M subscript will be deleted from G and similar expressions in the sequel). We point out that the existence of optimal estimates satisfying (1) with any of the $\alpha_{m_1 \ldots m_s} \neq 0$ for $s \geq 2$ would preclude the existence of any additive test model, including the logistic (Birnbaum, 1968), Rasch (Rasch, 1980), binomial (Huynh, 1977), and classical models.

Prior studies of nonadditive item scoring ~~methods~~ seem to stem from Horst's so-called configural scoring methods (Horst, 1954; Bussmeyer & Jones, 1983; Gaier & Lee, 1953; Jannarone & Roberts, 1984; McDonald, 1967). Configural scoring simply reflects nonadditivity in prediction by regressing criterion variables on raw items in a scale as well as on their cross-products. For example if there were an external criterion, Y, available in the Figure 1 case, then the configural approach would fit

$$E(Y|\underset{\sim}{x}) = \sum_{m=1}^{M} \alpha_m x_m + \sum_{m=1}^{M-1} \sum_{n=m+1}^{M} \alpha_{mn} x_m x_n, \qquad (2)$$

using regression methods, and ideally identify only α_{12} and α_3 through α_M as nonzero. Jannarone and Roberts (1984) used a configural variant to reflect nonadditivity in the absence of external criteria. They performed two-by-two analyses of variance to evaluate contributions of cross-products among each pair of items, but with the sum of the remaining M-2 item scores instead of an external variable as a criterion. Highly nonadditive item pairs were reflected by augmenting unit-weight scores with scores based on corresponding pairwise item cross-products, which are known as technical items (Fischer & Formann, 1982).

Configural scoring methods for binary items have not become popular, probably because they have not led to improved predictability under cross-validation. This problem is not only well known in external prediction (e.g. Wainwright, 1965) but also seems to hold in internal scale development (Jannarone & Roberts, 1984).

The failure of binary-item configural scores to cross-validate may be due to the lack of a justifying model for configural methods when items are binary. In particular, the only model that seems to justify configural scoring methods is a linear model with errors independent of observed scores. However, just as such models and methods have been found to be inappropriate for raw binary item scores, they seem to be inappropriate for binary item cross-products as well. For example, Jannarone and Roberts (personal communication) have found that their method leads to distorted conclusions for pairs of items that are very easy or very difficult, in line with similar problems that often occur when fitting linear models to binary items.

Rationale. So far it has been suggested that conjunctive processes should naturally be reflected through the use of item cross-products in latent trait estimates. Also, configural scoring methods seem to be the only methods developed up until now with provisions for such cross-products as a goal but with poor cross-validation as a result, evidently because they are not appropriate for binary data. Thus, configural scoring seems to be in the same state as the classical test model was before the development of item response theory. It seems natural to ask then, whether a viable set of binary item response models can be constructed that accommodate item cross-products. Given such models it should be possible to construct valid methods for identifying conjunctivity.

The models to be described in the sequel were developed with a justification for the inclusion of binary item cross-products as a goal. The basis that was ultimately selected for justifying item cross-products was their appearance in sufficient statistics for latent traits. For example, just as the Rasch kernel is based on the sum of item scores as a sufficient statistic, the conjunctive Rasch model to be described is based on a sufficient statistic involving the sum of item scores as well as item cross-product scores. Similarly, just as the logistic kernel is based on a weighted-sum sufficient statistic, the conjunctive logistic extension to be described is based on a weighted sum of item scores and cross-products. Out of several bases for establishing satisfactory conjunctive models that were pursued, sufficiency was by far the most successful, yielding models that retain many advantages of their additive counterparts, as will be shown.

The conjunctive models and methods be described have been designed to extend their compensatory (additive) counterparts, while including their strongest features. For example, the conjunctive Rasch extension, which will be described in detail, leads to:

(a) a variety of possible conjunctive models and corresponding sufficient statistics for θ--for example, in order to accommodate the Figure 1 case with M=4, one extended Rasch model could yield the sufficient statistic, $x_1x_2+x_3+x_4$, indicating that x_1 and x_2 have no unique additive contributions to θ, whereas another extended Rasch model could yield the sufficient statistic, $x_1x_2+x_1+x_2+x_3+x_4$, allowing x_1 and x_2 to have both additive and multiplicative contributions;

(b) item parameters that can be estimated independently of person parameters;

(c) consistent estimates of the item parameters;

(d) conditional maximum likelihood estimates of θ given the item parameters; and finally

(e) methods for testing different conjunctive models such as those implied by the sufficient statistics in (a).

Also, the conjunctive logistic extension, which will be briefly described, yields sufficient statistics of the form, $\Sigma\alpha_{m_1 \ldots m_s} x_{m_1} \ldots x_{m_s}$, as well as unique conditional maximum-likelihood ability parameter estimates. Thus, the conjunctive models as well as accompanying methods to be described have useful features along the lines of their additive counterparts.

From the viewpoint of efficient ability estimation, conjunctive methods may be seen as alternatives to the current test construction practice of deleting or replacing items that do not fit, say, an additive Rasch model. Conjunctive methods have the potential for considerable precision improvements, as can be seen from the technical item viewpoint (Fischer & Formann, 1982). Suppose for example that a pool of verbal analogies items behaved conjunctively, say with 10 items of the V type, 10 of the I type and 10 of the VI type. In this case standard item selection practice might lead to the retention of fewer than the 30 original items and as a result fewer than 31 distinct possible test scores, because many of the items would not fit a Rasch model. However, a conjunctive approach would not only retain all of original items, but also include 100 additional technical items obtained by multiplying each V item by each I item. Thus, it would be possible for individuals to have many more distinct test scores, given a conjuctive model, than 31, given a Rasch model. Consequently, conjuctively scored tests have the potential for increased precision. (On the other

hand, if the Rasch model held, attempts to include the 100 technical items would not increase precision at all.)

Some of the results to follow will show that such conjunctive structure may indeed be present in existing scales. These results will be given in terms of empirical tests for additivity among all possible item triplets in two selected measures. Other results will show the kind of precision increase that could be expected if conjunctive structure were properly reflected through a nonadditive sufficient statistic. These results will be in the form of maximum possible validity coefficients for certain nonadditive versus additive models. Details will be given in the sequel.

In practice, it would be useful not only to include the $\alpha_{m_1 \ldots m_s}$ instead of unit weights but also to identify the relative contributions of the individual components for each item. For example, if the Figure 1 case held for verbal analogies items, it seems likely that the relative contributions of V and I would vary over items. However, describing procedures for identifying such relative contributions are beyond this article's scope. In addition, only one of the simplest conjunctive extensions to be described, the conjunctive Rasch model, will be described in detail. Other conjunctive extensions will only be briefly introduced.

A specialized Rasch model developed by Fischer and Formann (1982) for describing Rorschach test data as well as Kempf's (1977) dynamic test model seem to be special cases of the conjunctive models to follow.

Conjunctive Rasch Models

The conjunctive Rasch kernel. One familiar form for the usual Rasch model (e.g. Andersen, 1980) is

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty}\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|\theta\}dG(\theta)$$

$$= \int_{-\infty}^{\infty} \prod_{m=1}^{M} \frac{\exp\{(\theta-\beta_m)x_m\}}{1+\exp\{(\theta-\beta_m)\}}dG(\theta), \qquad (3)$$

where $\underset{\sim}{x} = (x_1,\ldots,x_M)$ is a realization of $\underset{\sim}{X} = (X_1,\ldots,X_M)$, and $G(\theta)$ is the latent trait distribution function. We will use the following equivalent formulation, because it can be directly extended to reflect conjunctive structure:

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty} \nu(\theta)\exp\{\sum_{m=1}^{M} x_m(\theta-\beta_m)\}dG(\theta), \qquad (4)$$

where

$$\nu(\theta) = [\sum_{X}\exp\{\sum_{m=1}^{M} u_m(\theta-\beta_m)\}]^{-1},$$

and $X=\{\underset{\sim}{u}: u_m=0,1; m=1,\ldots,M\}$, the set of all 2^M possible score patterns. With either formulation it is easy to verify that local independence holds, that is for any $\theta\in(-\infty,\infty)$,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|\theta\} = \prod_{m=1}^{M}\Pr\{X_m=x_m|\theta\}. \qquad (5)$$

Our conjunctive extension replaces the Rasch kernel in (4) with

$$\Pr_R\{\underset{\sim}{X}=\underset{\sim}{x}|\theta\} = \nu_R(\theta)\exp\{\sum_R x_{m_1}\ldots x_{m_s}(\theta-\beta_{m_1\ldots m_s})\}, \qquad (6)$$

where

$$\nu_R(\theta) = [\sum_X \exp\{\sum_R u_{m_1}\ldots u_{m_s}(\theta-\beta_{m_1\ldots m_s})\}]^{-1}, \qquad (7)$$

and

$$R \subseteq G = \{(m_1,\ldots,m_s) \subseteq \{1,\ldots,M\};\ s=1,\ldots,M\}.$$

(We need to consider subsets of G because the kernel (6) is not identified when $R=G$ as we will show below.) As a first example, a conjunctive kernel corresponding to Figure 1 would have $R=\{(1,2),3,\ldots,M\}$. As a second example, two models that Fischer and Formann (1982, equations (16) and (17)) describe for Rorschach plate responses involves 10 pairs of subitems, (1a,1b,...,10a, 10b), such that each pair behaves conjunctively. The appropriate restricted summation set for that model would be $R=\{1a,1b,(1a,1b),2a,2b,(2a,2b),\ldots,10a, 10b,(10a,10b)\}$. Fischer and Formann motivate their model as an additive Rasch model involving 20 physical items as well as 10 technical items. (Restrictions of model (6) could be thought of in a similar way, once it is noticed that all physical items may not necessarily appear, whereas many other technical items may appear.)

We point out that conjunctive Rasch kernels violate local independence in general, even though the technical and physical items may be factored in the numerator of (6) into

$$\exp\{\sum_R x_{m_1}\ldots x_{m_s}(\theta-\beta_{m_1\ldots m_s})\} = \prod_R \exp\{x_{m_1}\ldots x_{m_s}(\theta-\beta_{m_1\ldots m_s})\}.$$

The local dependence is determined in the denominator of (6) through the functional dependence among the physical and technical items. For example, if $M=2$ and $R=\{1,2,(1,2)\}$ then

$$\nu_R(\theta) = \sum_{u_1=0}^{1} \sum_{u_2=0}^{1} \exp\{u_1(\theta-\beta_1)+u_2(\theta-\beta_2)+u_1u_2(\theta-\beta_{12})\}$$

$$= 1 + \exp\{\theta-\beta_1\} + \exp\{\theta-\beta_2\} + \exp\{(\theta-\beta_1) + (\theta-\beta_2) + (\theta-\beta_{12})\}$$

$$\neq \nu_1(\theta)\nu_2(\theta)\nu_{12}(\theta), \tag{8}$$

for any ν_1 and ν_{12}. As a result,

$$\Pr_R\{X_2=x_2|X_1=0;\theta\} = \frac{\exp\{(\theta-\beta_2)x_2\}}{1+\exp\{\theta-\beta_2\}},$$

but

$$\Pr_R\{X_2=x_2|X_1=1;\theta\} = \frac{\exp\{(2\theta-\beta_2-\beta_{12})x_2\}}{1+\exp\{2\theta-\beta_2-\beta_{12}\}} \neq \Pr_R\{X_2=x_2|X_1=0;\theta\}. \tag{9}$$

Given the conjunctive Rasch kernel (6), we have

$$\Pr_R\{\underset{\sim}{X}=\underset{\sim}{x}\} = \exp\{-\Sigma_R x_{m_1}\ldots x_{m_s}\beta_{m_1\ldots m_s}\}\int_{-\infty}^{\infty} \nu_R(\theta)\exp\{\theta t_R(\underset{\sim}{x})\}dG(\theta)$$

$$= \Pr_R\{\underset{\sim}{X}=\underset{\sim}{x}, T=t_R(\underset{\sim}{x})\}, \tag{10}$$

where

$$t_R(\underset{\sim}{x}) = \Sigma_R x_{m_1}\ldots x_{m_s}. \tag{11}$$

By the factorization theorem t_R is a sufficient statistic for θ. In addition, R can be chosen so that either t_R is additive, satisfying the Rasch model, or multiplicative, yielding a class of conjunctive/locally dependent models indexed by R. Finally, it is easy to verify that kernels satisfying (6)

are well-defined probability functions for all $R \subseteq G$ and real $\beta_{m_1 \ldots m_s}$. (For any $\underset{\sim}{x}$ the numerator is positive and the denominator is simply the sum of all possible numerators.)

(A potentially puzzling point should be clarified here. In most treatments of item response models, local independence is treated as an axiom, leading to kernels that are expressible as individual item probability products. In contrast, kernel (6) has been derived not from local independence but from the requirement that item cross-products appear in the sufficient statistic for θ. It only happens that the numerator of (6) can be analyzed into a product involving one factor for each physical and technical item. Fortunately, only the numerator of (6) can be factored into such a product, <u>not</u> the entire expression. Thus, (6) implies local dependence, in line with the cognitive dependence that should be associated with conjunctive processes as in Figure 1.)

The forms that item response functions take for conjunctive models are relatively complicated. Unlike the additive locally independent case (5), which leads to the simple and well-known item response functions,

$$\Pr\{X_m=1|\theta\} = 1/(1+\exp\{\theta-\beta_m\}\ ,\ m=1,..,\ M,$$

the conjuctive/locally dependent item response functions satisfying (6) cannot in general be simplified beyond the basic form,

$$\Pr\{X_m=1|\theta\} = v_R(\theta) \sum_{\substack{\underset{\sim}{x} \in X \\ x_m=1}} \exp\{ \Sigma_R x_{m_1} \ldots x_{m_s} (\theta - \beta_{m_1 \ldots m_s})\},\ m=1,\ldots,M.$$

However, for some simple conjunctive kernels such as in the above first (Figure 1) example, item response functions are correspondingly simpler. For that example, it can be shown that if $m=3,4,\ldots,M$, then

$$\Pr\{X_m=1|\theta\} = 1/(1+\exp\{\theta-\beta_m\}),$$

in line with the additive case, but if m=1 or 2 then

$$\Pr\{X_m=1|\theta\} = \frac{\exp\{\theta-\beta_m\}(1+\exp\{2\theta-\beta_{3-m}-\beta_{12}\})}{1+\exp\{\theta-\beta_{3-m}\} + \exp\{\theta-\beta_m\}(1+\exp\{2\theta-\beta_{3-m}-\beta_{12}\})}.$$

Estimation. This section follows developments by Andersen (Andersen, 1982) and Kelderman (Kelderman, 1984) for the Rasch model. (R subscripts are omitted in the sequel.) We have,

$$\Pr\{T=t\} = C(t)\int_{-\infty}^{\infty} \nu(\theta)\exp\{\theta t\}dG(\theta),$$

where

$$C(t) = \sum_{T} \exp\{-\sum_{R} x_{m_1}\ldots x_{m_s}\beta_{m_1}..m_s\}$$

and

$$T=\{\underset{\sim}{x}:\sum_{R} x_{m_1}\ldots x_{m_s}=t\}.$$

Thus from (9),

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|T=t\} = \frac{1}{C(t)}\exp\{-\sum_{R} x_{m_1}\ldots x_{m_s}\beta_{m_1}\ldots m_s\}, \quad \underset{\sim}{x} \in X, \qquad (12)$$

$$= 0 \text{ elsewhere.}$$

Equation (12) leads to consistent item parameter estimates and hypothesis tests, following known procedures for both additive Rasch model estimation (Rasch, 1980) and multinomial hypothesis testing (Landis & Koch, 1979). As an estimation example, if the model in Figure 1 held with M=4, then $t=x_1x_2+x_3+x_4$ would be a sufficient statistic for θ and the following equation would hold:

$$\begin{bmatrix} \log[\Pr\{\underset{\sim}{X} = (0,0,0,0)|T=0\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,0,0,0)|T=0\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,1,0,0)|T=0\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,0,1,0)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,1,0,0)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,1,1,0)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,0,1,0)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,1,0,1)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,0,0,1)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,0,0,1)|T=1\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,1,1,0)|T=2\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,1,0,1)|T=2\}] \\ \log[\Pr\{\underset{\sim}{X} = (1,0,1,1)|T=2\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,0,1,1)|T=2\}] \\ \log[\Pr\{\underset{\sim}{X} = (0,1,1,1)|T=2\}] \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0&0 \\ 1&0&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&1&0&0&1&0 \\ 0&1&0&1&0&0 \\ 0&1&0&0&1&0 \\ 0&1&0&0&1&0 \\ 0&1&0&0&0&1 \\ 0&1&0&0&0&1 \\ 0&1&0&0&0&1 \\ 0&0&1&1&1&0 \\ 0&0&1&1&0&1 \\ 0&0&1&0&1&1 \\ 0&0&1&0&1&1 \\ 0&0&1&0&1&1 \end{bmatrix} \begin{bmatrix} \chi(0) \\ \chi(1) \\ \chi(2) \\ \beta_{12} \\ \beta_3 \\ \beta_4 \end{bmatrix}, \quad (13)$$

where $\chi(j) = -\log[C(j)]$ $(j=0,1,2)$. (For this example there are four possible values of the sufficient statistic: t=0,1,2, and 3; t=3 is not included in (13) because only one possible pattern, (1,1,1,1), could yield t=3, hence $\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|T=3\}$ would be completely determined. Also, the design matrix in (13) is not of full rank, although this could easily be remedied through reparameterization.) Thus, consistent estimates for the item parameters can be easily obtained after replacing the left-hand side of (13) with logs of corresponding relative frequencies (although required sample sizes could be very large for some items). For some restricted kernels, more elegant estimation procedures seem possible, such as conditional maximum likelihood approaches following those of Andersen (Andersen, 1982, p. 245 ff.) for the additive Rasch kernel.

As (13) suggests, four-fold item contingencies can always be explained completely with enough conjunctive item parameters. Furthermore, it is easy to show that any M-fold item contingencies can be explained perfectly by some conjunctive Rasch model. In this sense conjunctive Rasch models are as general as would ever be necessary.

Equations of the form (13) can be easily used to show that Rasch conjunctive models with $R=G$ are unidentifiable. For example, 7 additive and technical item parameters would be involved when M=3 and $R=G$, although only 6 conditional probabilities could be obtained that are not necessarily 0 or 1. Thus, the 6 x 7 design matrix in the version of (13) with $R=G$ would have rank of at most 6 and consequently the 7 parameters could never be identified. Restricted conjunctive Rasch models may also be unidentifiable, especially when subsets of the total number of items must be used to estimate parameters, for example when some of the 2^M possible cells in an M-item frequency table are empty. In such instances graph-theoretic methods due to Fischer (1981) could be used to establish identifiability conditions and conjunctive models could be chosen accordingly.

For tests of even moderate length, the number of possible conjunctive item parameters can be prohibitively large, due to the small cell sizes expected in corresponding 2^M frequency tables. The problem can be alleviated by imposing theoretical restrictions on the structure, as in the above Rorschach example. Also, very simple conjunctive structures may be assumed, such as the involvement of pair-wise item cross-products only, and simple methods may be used to uncover them, such as looking only at three-way item contingencies to evaluate a given cross-product parameter. This latter approach will be illustrated in the following sections. Even so, the simple estimation and hypothesis testing procedures described here would have major defects for the global treatment of large test structures.

Turning next to ability estimation, we will simply point out that unique maximum likelihood estimates of θ exist, given the item parameters, under any conjunctive version of (6) and (7). The existence of such estimates for all t except t=0 and t=t(1,1,...,1) follows because the kernel (6) is a distribution from the one-parameter exponential family (see Andersen, 1980, Section 3.3). Given such unique estimates as well as the existence of unique item parameter estimates already illustrated, feasible ability estimation procedures are assured. (The argument parallels that of Birnbaum, 1968, p. 458).

Hypothesis testing. We will illustrate one of several possible approaches for which the latent trait distribution need not be specified. For any $R \subseteq G$ we have,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \Pr\{\underset{\sim}{X}=\underset{\sim}{x} \mid T=t(\underset{\sim}{x})\}\Pr\{T=t(\underset{\sim}{x})\}$$

$$= \frac{\exp\{-\sum_{R} x_{m_1}\ldots x_{m_s}\beta_{m_1\ldots m_s}\} \sum_{\underset{\sim}{u}\in T}\Pr\{\underset{\sim}{X}=\underset{\sim}{u}\}}{\sum_{\underset{\sim}{u}\in T}\exp\{-\sum_{R} u_{m_1}\ldots u_{m_s}\beta_{m_1\ldots m_s}\}} \qquad (14)$$

Turning again to the Figure 1 example, when M=3 and t=1 (14) yields

$$\Pr\{\underset{\sim}{X}=(0,0,1)\} - \frac{e^{-\beta_3}}{3e^{-\beta_3}+e^{-\beta_{12}}}[\Pr\{\underset{\sim}{X}=(0,0,1)\}+\Pr\{\underset{\sim}{X}=(1,0,1)\}+ \Pr\{\underset{\sim}{X}=(0,1,1)\}+\Pr\{\underset{\sim}{X}=(1,1,0)\}] = 0, \qquad (15a)$$

$$\Pr\{\underset{\sim}{X}=(1,0,1)\} - \frac{e^{-\beta_3}}{3e^{-\beta_3}+e^{-\beta_{12}}}[\Pr\{\underset{\sim}{X}=(0,0,1)\}+\Pr\{\underset{\sim}{X}=(1,0,1)\}+ \Pr\{\underset{\sim}{X}=(0,1,1)\}+\Pr\{\underset{\sim}{X}=(1,1,0)\}] = 0, \qquad (15b)$$

$$\Pr\{\underset{\sim}{X}=(0,1,1)\} - \frac{e^{-\beta_3}}{3e^{-\beta_3}+e^{-\beta_{12}}}[\Pr\{\underset{\sim}{X}=(0,0,1)\}+\Pr\{\underset{\sim}{X}=(1,0,1)\}+ \Pr\{\underset{\sim}{X}=(0,1,1)\}+\Pr\{\underset{\sim}{X}=(1,1,0)\}] = 0, \qquad (15c)$$

and

$$\Pr\{\underset{\sim}{X}=(1,1,0)\} - \frac{e^{-\beta_{12}}}{3e^{-\beta_3}+e^{-\beta_{12}}}\left[\Pr\{\underset{\sim}{X}=(0,0,1)\}+\Pr\{\underset{\sim}{X}=(1,0,1)\}+\Pr\{\underset{\sim}{X}=(0,1,1)\}+\Pr\{\underset{\sim}{X}=(1,1,0)\}\right] = 0, \tag{15d}$$

When t=0 (14) yields

$$\Pr\{\underset{\sim}{X}=(0,0,0)\} = \tfrac{1}{3}[\Pr\{\underset{\sim}{X}=(0,0,0)\}+\Pr\{\underset{\sim}{X}=0,1,0)\}+\Pr\{\underset{\sim}{X}=(1,0,0)] \tag{15e}$$

$$= \Pr\{\underset{\sim}{X}=(0,1,0)\} \tag{15f}$$

$$= \Pr\{\underset{\sim}{X}=(1,0,0)\}. \tag{15g}$$

Now since consistent estimates of β_{12} and β_3 exist, standard methods can be used to test equations (15) by simultaneously evaluating any exhaustive, linearly independent set of resulting functions. (e.g., Koch & Landis, 1977). For example, (15) may be tested by evaluating

$$H_o: \underset{5x1}{\underset{\sim}{f}}\ (\underset{7x1}{\underset{\sim}{p}}) = 0,$$

where $f_1(\underset{\sim}{p})$ through $f_3(\underset{\sim}{p})$ are the left-hand sides of (15a) through (15c), respectively, $f_4(\underset{\sim}{p})=\Pr\{\underset{\sim}{X}=(0,0,0)\}-\Pr\{\underset{\sim}{X}=(0,1,0)\}$, $f_5(\underset{\sim}{p})=\Pr\{\underset{\sim}{X}=(0,1,0)\}$ $-\Pr\{\underset{\sim}{X}=(1,0,0)\}$, and $\underset{\sim}{p}^T=[\Pr\{\underset{\sim}{X}=(0,0,1)\}, \Pr\{\underset{\sim}{X}=(1,0,1)\}, \Pr\{\underset{\sim}{X}=(0,1,1)\}$, $\Pr\{\underset{\sim}{X}=(1,1,0)\}, \Pr\{\underset{\sim}{X}=(0,0,0)\}, \Pr\{\underset{\sim}{X}=(0,1,0)\}, \Pr\{\underset{\sim}{X}=(1,0,0)\}]$, where ^{T} denotes transposition. (In the $f_j(\underset{\sim}{p})$ β_{12} and β_3 are replaced with appropriate functions of $\underset{\sim}{p}$ based on an expression like (13).) The test statistic is $\underset{\sim}{f}(\hat{\underset{\sim}{p}})^T S_{\underset{\sim}{f}}^{-1} \underset{\sim}{f}(\hat{\underset{\sim}{p}})$, where $\hat{\underset{\sim}{p}}$ is the vector of relative frequencies corresponding to $\underset{\sim}{p}$, and

$$S_{\underset{\sim}{f}} = \frac{1}{N_{\hat{\underset{\sim}{p}}}}\left[\frac{\partial f_i}{\partial p_j}\bigg|_{\underset{\sim}{p}=\hat{\underset{\sim}{p}}}\right]^T\left[\hat{\underset{\sim}{p}}\hat{\underset{\sim}{p}}^T - \underset{\sim}{D}_{\hat{\underset{\sim}{p}}}\right]\left[\frac{\partial f_i}{\partial p_j}\bigg|_{\underset{\sim}{p}=\hat{\underset{\sim}{p}}}\right],$$

where $(\underset{\sim}{D}_{\hat{p}})_{ii}=\hat{p}_i$, $(\underset{\sim}{D}_{\hat{p}})_{ij}=0$ $(i\neq j; j=1,\ldots,7)$, and $N_{\hat{p}}$ is the total observed frequency associated with the relative frequencies in $\hat{p}$. ($N_{\hat{p}}$ is the same as overall sample size unless some individuals have some missing item responses.) The sampling distribution for the test statistic is asymptotically chi square with 5 degrees of freedom (as sample size increases--sample size should be sufficiently large that $\underset{\sim}{f}$ is approximately 5-variate normal).

Other conjunctive and additive Rasch models may be tested similarly.

Potential conjunctive model utility. We will briefly illustrate the potential for improved ability estimation, the potential for uncovering conjunctive structure, and some preliminary evidence that test data are nonadditive.

Several indices of estimator precision could be used to compare conjunctive with compensatory model performance. These include analytic as well as Monte Carlo measures of ability-estimator squared error, efficiency, bias, and how precisely the estimators rank-order individuals with respect to latent traits. We have chosen an analytic measure of rank-ordering precision, namely the product-moment correlations of latent traits with Bayes estimates (based on squared-error-loss), equivalent to the validity (Lord & Novick, 1968) of Bayes estimators. (The Bayes estimates to be evaluated are based on prespecified, discrete latent trait distributions defined on a small set of points.) We chose this measure because Bayes estimate-latent trait correlations are the best possible validity coefficients for any model (Rao, 1965, p. 265), hence useful global indices of optimal test efficiency. In addition, these correlations may be nearly achievable in practice, given the large test sizes available in educational

testing applications. Finally, Bayes estimates and their validities can be easily machine-computed and evaluated (given a discrete prior on a small point set) without using Monte Carlo methods.

Returning once again to the Figure 1 case, we will compare Bayes validities from three kernels, each based on $M=M_V+M_I+M_{VI}$ items. The compensatory comparison kernel has a sufficient statistic of the form,

$$\sum_{m=1}^{M_V+M_I+M_{VI}} x_m \tag{16}$$

and equivalent items with zero-valued parameters. The two conjunctive kernels to be compared both involve three equivalent item sets--M_V items of the V type, M_I items of the I type and M_{VI} of the global type--along with zero-valued item parameters. The first conjunctive kernel has the sufficient statistic,

$$\sum_{m=1}^{M_V} \sum_{n=M_V+1}^{M_V+M_I} x_m x_n + \sum_{m=M_V+M_I+1}^{M_V+M_I+M_{VI}} x_m , \tag{17}$$

and the second conjunctive kernel has the sufficient statistic,

$$\sum_{m=1}^{M_V} \sum_{n=M_V+1}^{M_V+M_I} x_m x_n + \sum_{m=1}^{M_V+M_I+M_{VI}} x_m . \tag{18}$$

Kernel (18) extends (17) to include items that are both compensatory and conjunctive. Equation (18) is considerably less restrictive than (17) and more in line with the configural model (2).

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Insert Table 1 about here

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Table 1 gives validity comparisons for kernels (16), (17), and (18), six different latent trait distributions, and three scale size configurations: $M_V=M_I=M_{VI}=5$, 10, and 15. The results range from much higher conjunctive model validity than additive model validity for low-variance latent trait distributions to slightly higher additive model validity for the highest-variance distribution.

In evaluating the potential for uncovering conjunctive structure, we will provide performance data from hypothesis tests that are similar to the test illustrated in the previous section. The tests evaluate two prespecified structures among a pair of items, labelled items 1 and 2 below: additive pair-wise structure satisfying (16) and purely conjunctive pair-wise structure satisfying (17). Although only the structure between items 1 and 2 will be of interest, a third item, labelled item 3 below, will be included in order to resolve the two structures empirically.

The test for pair-wise additivity evaluates kernels of the form,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|\theta\} = \nu(\theta)\exp\{\sum_{m=1}^{3} x_m(\theta-\beta_m)\}, \tag{19}$$

whereas the test for pure pair-wise conjunctivity evaluates kernels of the form,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}|\theta\} = \nu_R(\theta)\exp\{x_1x_2(\theta-\beta_{12})+x_3(\theta-\beta_3)\}. \tag{20}$$

To the extent that these two tests reject kernels when appropriate, it should be possible to partition pairs of scale items into the V, I, and VI types suggested by Figure 1. However, additive and conjunctive kernels for pairs of items satisfying (18) cannot be directly tested using item triplets, because (18) requires more estimated parameters than three-way item contingencies can yield. Even so, if both kernels (19) and (20) were rejected an additive and conjunctive kernel satisfying (18) would be implied.

Each entry in Table 2 gives correct rejection proportions among 100 .05-level tests. The first two columns correspond to tests for the additivity associated with (19), given either conjunctivity and additivity as in (18) or pure conjunctivity as in (20). The last two columns correspond to tests for the conjunctive kernel (20), given either pure additivity as in (19) or conjunctivity and additivity as in (18). Rows correspond to different latent trait distributions as in Table 1, and each entry in parentheses gives correct rejection proportions among 100 tests based on 1,000, 5,000, and 10,000 simulated observations, from left to right.

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Insert Table 2 about here

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Table 2 shows that even 10,000 observations are not enough to yield high correct rejection rates when latent trait distributions have low variances, especially for the additivity test. However, correct rejection rates are very high when latent trait distributions have high variances. This makes sense because individuals having extreme latent trait values will tend to show much different test patterns for the three models than individuals with less extreme values. Overall Type I error rates for the tests of (19) and (20) were obtained by noting whether or not the data for columns one and two of Table 2 led to rejections of models (19) and (20), respectively. These Type I error rates were .047 and .054, respectively.

As the results in Table 2 clearly show, the tests for resolving items into additive and conjunctive subsets could lead to a substantial proportion of incorrect item classifications. Moreover, the incorrect classification rates would be even higher if more complicated conjunctive structures existed involving more than two component processes. These problems

may be lessened if enough correct classifications can be obtained in practice to suggest a theoretical basis for grouping items, given their cognitive content. For example, if the most significant items included in an empirically determined analogical reasoning component clearly contained a pure vocabulary element, correct decisions could be made about other items that were less significant based on their vocabulary content. Whether such theoretical cues will in fact aid in correctly resolving items remains to be seen. (Of course, matters would be simpler from a methodological viewpoint if cognitive theory suggested certain structures at the outset and the above methods were used only to check them--practitioners, on the other hand, might welcome these methods as exploratory aids in uncovering conjunctive structures.)

Although detailed empirical results are beyond this article's scope, two preliminary results seem promising. We have performed additivity tests for all item triplets based on 1,000 scores from a 50-item verbal analogies test (from the Spring, 1970 normative sample of the Level 3, Form A, Verbal School and College Ability subtest, provided by Educational Testing Service) as well as a 42-item personality scale (the Responsibility Scale from the CPI (Megargee, 1972)--respondents were high school males). Among the respective 19,600 and 11,480 triplet tests performed at the .01 level, 4,378 or 37% of the verbal analogies results were significant and 2,092 or 18% of the personality results were significant. Since Rasch nonadditivity far exceeded chance levels in these data, it seems that fitting conjunctive models could improve trait estimation efficiency. Moreover, for scales such as these, where there is reason to expect some conjunctive structure, fitting simple conjunctive models may be a useful alternative to the current practice of deleting items that do not fit a given additive model.

Other Conjunctive Kernels

In this section, conjunctive extensions will be first be given for three other compensatory item response models: the binomial model,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_0^1 [\prod_{m=1}^{M} \pi^{x_m}(1-\pi)^{1-x_m}]dH(\pi),$$

the logistic model,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty} \prod_{m=1}^{M} \frac{\exp\{x_m\alpha_m(\theta-\beta_m)\}}{1+\exp\{x_m\alpha_m(\theta-\beta_m)\}}dS(\theta),$$

and the general item response model,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty} \prod_{m=1}^{M} [\tau_m(\theta)]^{x_m}[1-\tau_m(\theta)]^{1-x_m}dU(\theta),$$

where the monotone increasing item response functions, $\tau_m(\theta)$, satisfy $0<\tau_m(\theta)<1$. The conjunctive extensions are respectively,

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_0^1 \gamma(\pi)\prod_R \pi^{x_{m_1}\cdots x_{m_s}}dH(\pi), \tag{21}$$

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty} \delta(\theta)\exp\{\sum_R \alpha_{m_1\cdots m_s} x_{m_1}\cdots x_{m_s}(\theta-\beta_{m_1\cdots m_s})\}dS(\theta), \tag{22}$$

and

$$\Pr\{\underset{\sim}{X}=\underset{\sim}{x}\} = \int_{-\infty}^{\infty} \varepsilon(\theta)[\prod_R \tau_{m_1\cdots m_s}(\theta)^{x_{m_1}\cdots x_{m_s}}]dU(\theta), \tag{23}$$

where γ, δ and ε are normalizing functions and H, S, and U are distribution functions, analogous to ν and G respectively, for the conjuctive Rasch model.

Among these conjunctive kernels the binomial with $R=G$ is the least interesting, because the values of its sufficient statistic are in one-to-one correspondence with the usual number-correct binomial statistic, hence it has no potential for improved precision over the usual binomial model. However, restricted versions are more useful as for example in (17) and (18), which are actually restricted conjuctive binomial kernels.

For the conjunctive logistic kernel, similar arguments to those given earlier for the Rasch conjunctive model identify

$$\sum_{R} \alpha_{m_1 \cdots m_s} x_{m_1} \cdots x_{m_s} \qquad (24)$$

as a sufficient statistic for θ (given the $\alpha_{m_1 \cdots m_s}$). Functionally, the $\alpha_{m_1 \cdots m_s}$ can provide for differential weights in item cross-product terms as well as raw item terms. However, the logistic extension has the same liabilities relative to the Rasch extension as does the logistic model relative to the Rasch model. For example, the distribution of $\underset{\sim}{X}$ given t is not independent of θ for the logistic conjunctive kernel.

Finally, it may be shown that the above four discrete conjunctive kernels follow the same generality hierarchy as their compensatory counterparts. That is, for the following ordering of conjunctive kernels--(1) binomial, (2) Rasch, (3) logistic and (4) general--each among models (1) through (3) is a special case of its successors.

Conjunctive extensions of kernels for the classical test theory model may also be constructed, as will be shown next. Using the preceding notation, the classical model requires that $\underset{\sim}{X}$ have conditional expectation $E\{\underset{\sim}{X}|\theta\}=\sigma^2_e \underset{\sim}{1}_M$, where σ^2_e is the classical model item error variance and $\underset{\sim}{1}_M$ is an M-vector of ones.

Now it is well known that the classical model requirements are satisfied when $\underset{\sim}{X}$ is continuous and its M-variate kernel density has the normal form,

$$f\{\underset{\sim}{x}|\theta\} = \zeta(\theta,\sigma_e^2)\exp\{-\tfrac{1}{2}(\underset{\sim}{x}-\underset{\sim}{1}_M\theta)^T\underset{\sim}{V}_{\underset{\sim}{x}}^{-1}(\underset{\sim}{x}-\underset{\sim}{1}_M\theta)\}, \qquad (25)$$

where $\underset{\sim}{V}_{\underset{\sim}{x}}=\underset{\sim}{I}_M\sigma_e^2$, $\underset{\sim}{I}_M$ is the (MxM) identity matrix, and

$$\zeta(\theta,\sigma_e^2) = [\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}\exp\{-\tfrac{1}{2}(\underset{\sim}{u}-\underset{\sim}{1}_M\theta)^T\underset{\sim}{V}_{\underset{\sim}{x}}^{-1}(\underset{\sim}{u}-\underset{\sim}{1}_M\theta)\}du_1\ldots du_M]^{-1}.$$

Also, it can be shown that if (25) holds then $x_1+\ldots+x_M$ is sufficient for θ. We introduce the following general conjunctive extension of the normal kernel (25). For $\underset{\sim}{w}^T=(x_1,\ldots,x_M,x_1x_2,\ldots,x_1x_2\ldots x_M)$, let

$$g\{\underset{\sim}{x}|\theta\} = \zeta(\theta,\sigma_e^2)\exp\{-\tfrac{1}{2}(\underset{\sim}{w}-\underset{\sim}{1}_{2^M}\theta)^T\underset{\sim}{V}_{\underset{\sim}{w}}^{-1}(\underset{\sim}{w}-\underset{\sim}{1}_{2^M}\theta)\},$$

where $\underset{\sim}{V}_{\underset{\sim}{w}}=\underset{\sim}{I}_{2^M}\sigma_e^2$ and

$$\zeta(\theta,\sigma_e^2) = [\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}\exp\{-\tfrac{1}{2}(\underset{\sim}{t}-\underset{\sim}{1}_{2^M}\theta)\underset{\sim}{V}_{\underset{\sim}{w}}^{-1}(\underset{\sim}{t}-\underset{\sim}{1}_{2^M}\theta)\}du_1\ldots du_M]^{-1},$$

where $\underset{\sim}{t}^T=(u_1,\ldots,u_M,u_1u_2,\ldots,u_1u_2\ldots u_M)$. It may be shown that if (26) holds $g(\underset{\sim}{x}|\theta)$ is a <u>bona fide</u> probability density; $x_1+\ldots+x_M+x_1x_2+\ldots+x_1x_2\ldots x_M$ is sufficient for θ; the items are locally dependent (note that ζ cannot be analyzed into physical and technical item factors due to functional dependencies among the w_m, just as v cannot be so factored for the Rasch case--the two kernels are thus very closely related); and although (26) may not be adequately identified as it stands (as with the unrestricted conjunctive Rasch kernel) it may be restricted as in the previously described

discrete cases to yield sufficient statistics of the form (11)--in particular if any subvector of $\underset{\sim}{w}$ and corresponding submatrix of $\underset{\sim}{\underset{\sim}{V}}_{\underset{\sim}{w}}$ replaces $\underset{\sim}{w}$ and $\underset{\sim}{\underset{\sim}{V}}_{\underset{\sim}{w}}$ respectively in (26), then terms in the sufficient statistic for θ will correspond to the elements retained in the subvector. Also, (26) may be extended to include weighted-sum sufficient statistics of the form (24) by replacing $\underset{\sim}{\underset{\sim}{V}}_{\underset{\sim}{w}}$ with a nondiagonal positive definite matrix. Finally, estimation and hypothesis testing seem feasible given the similarity of (26) to the multivariate normal kernel and the fact that (26) belongs in the multiparameter exponential family.

Conclusion

Future directions. Conjunctive model aspects requiring more work include resolving identifiability and capitalizing-on-chance problems, developing efficient estimation procedures such as conditional maximum likelihood for the Rasch extensions, formulating extensions with guessing provisions for multiple choice tests, and determining whether these models are useful in practice. Conjunctive models might also be used to resolve certain issues regarding dimensionality, item response function form, and local dependence. For example, all five kernels presented seem to imply that a model may be both unidimensional and locally dependent, in contrast to some seemingly sensible assertions (Andrich, 1984). Finally, a closer look at the conjunctive kernels that have only been briefly mentioned here could be useful. For example, it seems likely that the conjunctive extensions of the normal ogive and Thurstonian choice models could be based on variants of kernel (26), or its continuous logistic analog (Yellot, 1977). Such extensions could provide some needed flexibility in the available models for choice data. Thus, with the continuous as well as discrete kernels presented, many questions remain unanswered.

Summary. First, an item response theory has been introduced that includes multiplicative sufficient statistics based on locally dependent models, which have a conjunctive cognitive structure interpretation. Second, it has been shown that consistent estimation and ad hoc hypothesis testing procedures exist for conjunctive models and that more efficient methods seem possible. Third, these ad hoc procedures have been used to show that conjunctive models may have promise for constructing more efficient tests and uncovering conjunctive structure in large samples. Finally, it is clear that major work remains before concluding whether or not conjunctive models are useful in practice.

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Table 1

Validities of Bayes Estimators for Additive and Conjunctive Rasch Models ($\times 10^2$)

Latent Trait Distributions[a]	Kernels: Additive(16)[b]	Conjunctive(17)	Conjunctive and additive (18)
Quasinormal			
(.75)	(59,72,79)[c]	(83,93,95)	(85,95,97)
(1.5)	(82,89,93)	(91,95,96)	(93,96,97)
(3.0)	(93,97,98)	(94,96,97)	(96,97,97)
Uniform			
(.75)	(72,83,88)	(91,96,96)	(92,97,98)
(1.5)	(90,95,96)	(94,96,97)	(96,97,97)
(3.0)	(96,98,99)	(95,97,97)	(96,97,98)

a Latent trait distributions are discrete with positive probability on 5 points: $-\theta$, $-\theta/2$, 0, $\theta/2$ and θ. Numbers in parentheses below this heading are values of θ. Quasinormal distributions assign probabilities of .07, .24, .38, .24, and .07 respectively to the 5 points, whereas uniform distributions assign .2 to each point.

b The numbers to the right of kernel labels are the text equations that describe the kernels fully.

c Each triplet gives Bayes estimate-latent trait correlations based, from left to right, on 5, 10, and 15 items of each type (M_V, M_I, and M_{VI} in equations 16, 17, and 18).

Table 2

Correct Rejection Percentages for Additive and Conjunctive Tests (among 100 .05 level tests for each entry).

Latent Trait Distributions[a]	Test for Additive Model(16) When the True Model is (17)	Test for Additive Model(16) When the True Model is (18)	Test for Conjunctive Model(17) When the True Model is (16)	Test for Conjunctive Model(17) When the True Model is (18)
Quasinormal				
(.75)	(4, 9, 9)[b]	(8,22, 44)	(25, 70, 93)	(17, 84, 99)
(1.5)	(13, 35, 66)	(20,83,100)	(92,100,100)	(98,100,100)
(3.0)	(47,100,100)	(30,98,100)	(100,100,100)	(100,100,100)
Uniform				
(.75)	(5, 11, 23)	(11,46, 76)	(30,100,100)	(70,100,100)
(1.5)	(33, 98, 99)	(49,98,100)	(100,100,100)	(100,100,100)
(3.0)	(91,100,100)	(59,99,100)	(100,100,100)	(100,100,100)

a Latent trait distributions are discrete with positive probability on 5 points: $-\theta$, $-\theta/2$, 0, $\theta/2$ and θ. Numbers in parentheses below this heading are values of θ. Quasinormal distributions assign probabilities of .07, .24, .38, .24, and .07 respectively to the 5 points, whereas uniform distributions assign .2 to each point.

b Each triplet gives correct rejection percentages for .05 level chi-square tests from simulations based, from left to right, on 1000, 5000, and 10,000 observations.

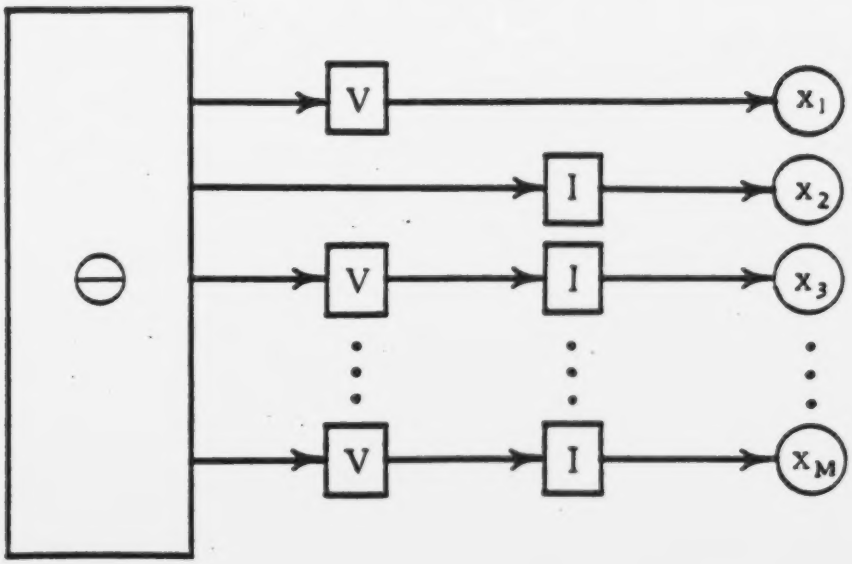

Figure 1

A Simple Conjunctive Model

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